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Heat transfer by natural convection from the inside surface of a uniformly heated tube at different angles of inclination

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Abstract—Natural convection from the inside surfaces of vertical and inclined tubes to air is experimentally investigated in the ranges of $Gr_{ml}\, Pr$ from 1.44×10^7 to 8.85×10^8 and θ from 0° to 75°. The results obtained are correlated by dimensionless groups and compared with the available data. A length to diameter ratio from 10 to 31.4 had practically no effect on the mean heat transfer from the whole tube in the range of $Gr_{ml}\, Pr$ employed.

1. INTRODUCTION

THE AVAILABLE work on natural convection from the inside surfaces of tubes open at both ends is very limited. It is only recently that this case has attracted attention due to the use of this tube in practical fields such as solar collectors and nuclear engineering.

However, most of the available investigations are theoretical and deal with the vertical tube only. To the authors' knowledge no work is available on inclined tubes.

The present work has been carried out in an attempt to fill a part of the existing gap. It provides experimental data on uniformly heated (constant flux) vertical and inclined tubes.

2. PREVIOUS WORK

A brief description of previous work on uniformly heated (constant flux) vertical tubes is hereinafter given. No work was found on inclined tubes. Investigations in the field of isothermal tubes are not included.

Bennet [1], in 1943, carried out an experimental investigation on vertical tubes placed in air. As found by Ede [2], the range of $Gr_{ml}\, Pr$ varied from 4×10^7 to 1.1×10^8 and the results were very scanty.

Al-Arabi [3], in 1955, experimented with a tube placed in air in the range of $Gr_{ml}\, Pr$ from 8×10^8 to 2.5×10^9. A relation between Nu_{ml} and $Gr_{ml}\, Pr$ was obtained.

Ede [2], in 1956, examined and compared the available data on vertical tubes. With the exception of Bennet's data [1] all the tubes were isothermal.

Kageyama and Isumi [4], in 1970, Davis and Perona [5], in 1971, and Dyer [6], in 1975, employing a finite difference technique, solved the boundary layer governing equations and obtained relations between the different variables. Of special interest were the relations given by Dyer for Nu_{mr} against $Gr_{mr}\, Pr(r/l)$ as they can be compared directly with experimental results.

Ismaeel [7], in 1977, carried out experimental work on tubes placed in air in the range of $Gr_{ml}\, Pr$ from 4.2×10^7 to 1.02×10^{10}. A relation between Nu_{ml} and $Gr_{ml}\, Pr$ was obtained.

3. APPARATUS

The apparatus used is essentially a brass tube with both ends open. The tube is carried on a frame which can be tilted to give any angle of inclination and is heated uniformly.

Five tubes, the particulars of which are given in Table 1, were employed and the experiments were carried out at angles of inclination equal to 0° (the vertical position), 30°, 45°, 60° and 75°.

As shown in Fig. 1, the tube (1) is covered with glass cloth (2) which acts as an electric insulator on which a heater (3) (the main heater) is uniformly wound. The heater is covered with a layer of asbestos (4) of about 70 mm thickness. On the asbestos a guard heater (5) is wound. Three pairs of thermocouples (6) (guard heater thermocouples) are inserted in the asbestos (4) along the tube. The thermocouples of each pair lie on the same line radially. The input to the guard heater is adjusted so that, at steady state, the readings of the two thermocouples of each pair

Table 1. Particulars of tubes used

Tube No.	Length, l (mm)	Inside diameter, d (mm)	l/d
1	197	19.7	10
2	334.9	19.7	17
3	453.1	19.7	23
4	565	19.7	28.7
5	423.5	13.5	31.4

NOMENCLATURE

A	constant
C	constant
d	tube inside diameter
f	correction factor (equation (7))
g	gravity acceleration
Gr	Grashof number
h	heat transfer coefficient
k	thermal conductivity
l	tube length
n	constant
Nu	Nusselt number
Pr	Prandtl number
q	heat flux
r	tube radius
t	temperature
x	axial distance from tube entrance.

Greek symbols

β	coefficient of volumetric expansion
θ	angle of inclination
μ	viscosity
ρ	density.

Subscripts

ent	entrance
i	inlet
m	mean
mf	mean film
ml	mean based on tube length
mr	mean based on tube radius
ms	mean surface
s	surface.

become practically equal. All the input to the main heater is then flowing to the air inside the tube.

The tube surface temperature is measured by calibrated glass-covered copper–constantan thermocouples (7) soldered in slots milled in the outside surface. As a result of the high thermal conductivity of brass the drop of temperature through the wall is negligible and the readings taken are considered equal to the inside surface temperature. The leads of the thermocouples are insulated by glass sleeves to minimize conduction to the hot junction through them.

In order to check the uniformity of temperature around the circumference, additional thermocouples are soldered to the tube opposite to the surface thermocouples at three positions along the tube. In all the inclined tubes used the readings of each of the surface and the opposite additional thermocouples were practically the same, a fact which may be attributed to the high thermal conductivity of the tube metal.

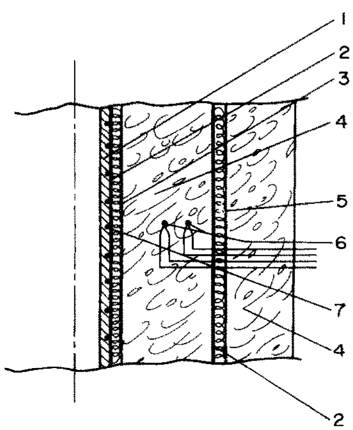

FIG. 1. General arrangement of apparatus: 1, tube; 2, electric insulation; 3, main heater; 4, asbestos layer; 5, guard heater; 6, guard heater thermocouples; 7, surface thermocouples.

The readings of the thermocouples were taken by means of a potentiometer capable of reading to 0.01 mV and the input to the main heater was recorded by an accurate wattmeter. The ambient temperature was measured by a calibrated mercury-in-glass thermometer capable of reading to 0.1°C.

To make sure that the surface thermocouples were properly soldered to the tube, a check was carried out by taking the readings with steam passing in the tube.

4. RESULTS

To calculate the coefficient of heat transfer to a fluid flowing inside a tube the difference between the surface temperature and the fluid bulk temperature is needed. The bulk temperature is, however, unknown in the present case. Therefore, a heat transfer coefficient based on the difference between the surface temperature and the fluid temperature at the entrance is normally used [5, 6]. The mean heat transfer coefficient thus calculated will be

$$h_m = q/t_{ms} - t_i \tag{1}$$

where

$$t_{ms} = \frac{l}{L}\int_0^l t_s \, dx. \tag{2}$$

Based on the tube length, the corresponding mean Nusselt number will be

$$Nu_{ml} = h_m L/k \tag{3}$$

and the corresponding Grashof number will be

$$Gr_m = g\beta(t_{ms} - t_i)l^3\rho^2/\mu^2. \tag{4}$$

The physical properties in this work are taken at a mean film temperature as given by

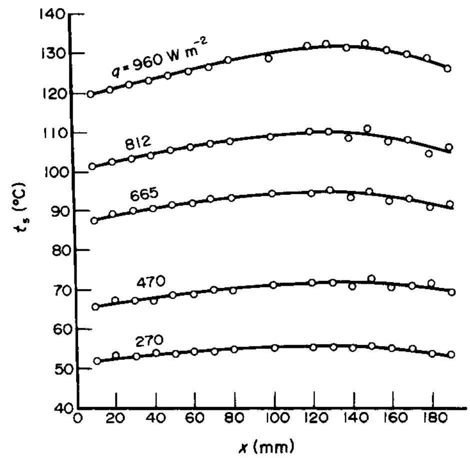

FIG. 2. Variation of t_s for tube No. 1 for different heat fluxes ($\theta = 0°$).

$$t_{mf} = (t_{ms} + t_i)/2. \quad (5)$$

4.1. *Variation of surface temperature with tube length*

The variation of the surface temperature t_s with tube length is shown in Fig. 2 for one of the vertical tubes at different fluxes. The t_s–x curves for all the other tubes have the same general shape. The value of t_s gradually increases with length until a limit beyond which it begins to decrease.

This phenomena, which was also noted in refs. [2, 7] can be explained if Fig. 3 is considered. At the entrance to the tube (point a) the thickness of the boundary layer is zero. Then it gradually increases until, at point b, the boundary layer fills the tube. From point a to point b the heat transfer gradually decreases and t_s gradually increases.

Beyond point b one would expect a straight line t_s–x relation (bc) the case being that of constant flux. However, as the air is heated along the tube, its physical properties gradually change with the increased temperature. The thermal conductivity increases causing less resistance to the flow of heat and the viscosity increases causing radial flow of the hotter layers of air nearer to the surface to the tube centre. A gradual increase of the local heat transfer beyond point b must then result. For constant flux this can only take place if the local difference between the bulk air temperature

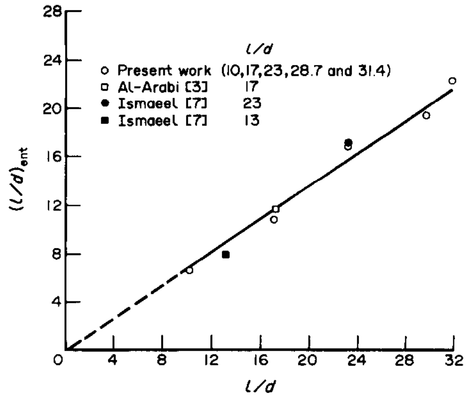

FIG. 4. Variation of entrance length with l/d.

(as shown by straight line ac″) and the surface temperature decreases resulting in the shape of the t_s–x curve (abc′) shown.

4.2. *Entrance length*

The length AB (Fig. 3) required for the boundary layer to fill the tube will hereinafter be called the 'entrance length', $(l/d)_{ent}$. Although one would expect $(l/d)_{ent}$ to depend on $Gr_{ml}\,Pr$, the experimental t_s–x curves for the same tube show practically the same value of $(l/d)_{ent}$ irrespective of the value of $Gr_{ml}\,Pr$ or inclination apparently due to the insufficiency of the range of $Gr_{ml}\,Pr$ employed to give measurable differences.

Plotting $(l/d)_{ent}$ for the different tubes against l/d gives a straight line as shown in Fig. 4. The equation of this line is

$$(l/d)_{ent} = 0.688(l/d). \quad (6)$$

The values of $(l/d)_{ent}$ read from the t_s–x curves of refs. [2, 7] are also plotted in Fig. 4. As can be seen they are in good agreement with the present results.

4.3. *Results of vertical tubes*

The values of Nu_{ml} for vertical tubes are plotted logarithmically in Fig. 5 against $Gr_{ml}\,Pr$. The dashed line is a straight line with a slope of 0.25. It will be seen that the $Nu_{ml}/(Gr_{ml}\,Pr)$ relation for all the tubes has the same general shape. Nu_{ml} increases gradually

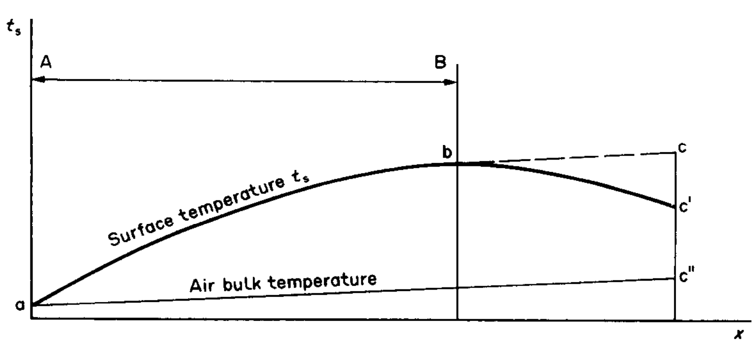

FIG. 3. General shape of temperature variation along the tube.

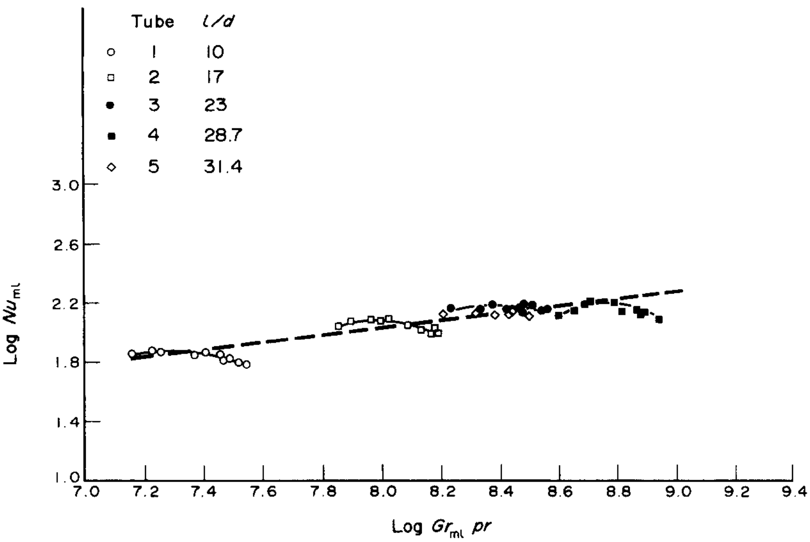

FIG. 5. Variation of Nu_{ml} with Gr_{ml} Pr for vertical tubes.

with Gr_{ml} Pr at a rate which gradually decreases until a limit beyond which the rate of increase becomes negative and Nu_{ml} begins to decrease with Gr_{ml} Pr. The same shape of curve was also obtained in refs. [2, 7].

This phenomena is believed to be a result of the radial variation of physical properties caused by the temperature gradient across the air, an effect which, as suggested by Kays and London [8], may be covered by means of a factor which depends on the air bulk temperature and the tube surface temperature. As the bulk temperature of air is normally unknown in the present case, the inlet air temperature was used and a factor f as given by equation (7), was employed

$$f = 1+0.05(t_{ms}-t_i)/t_i. \tag{7}$$

A plot of log ($Nu_{ml}\,f$) against log (Gr_{ml} Pr) is shown in Fig. 6. The results treated in this way, as can be seen, are represented by an equation of the form

$$Nu_{ml} = \frac{A}{f}(Gr_{ml}\,Pr)^n. \tag{8}$$

The equation for vertical tubes (with $A = 1.11$ and $n = 0.25$) then becomes

$$Nu_{ml} = \frac{1.11}{[1+0.05(t_{ms}-t_i)/t_i]}(Gr_{ml}\,Pr)^{0.25}. \tag{9}$$

Equation (9) is valid in the ranges of Gr_{ml} Pr from 1.44×10^7 to 0.85×10^8 and l/d from 10 to 31.4.

In fact the range of Gr_{ml} Pr in which equation (9) may be used can safely be extended to Gr_{ml} $Pr = 1.02 \times 10^{10}$ if the results of other observers, which are in good agreement with the present results as shown under Section 5, are taken into consideration.

The fact that the value of n in equation (9) is 0.25 shows that all the experiments carried out were in the laminar range. The onset of turbulence in the case of

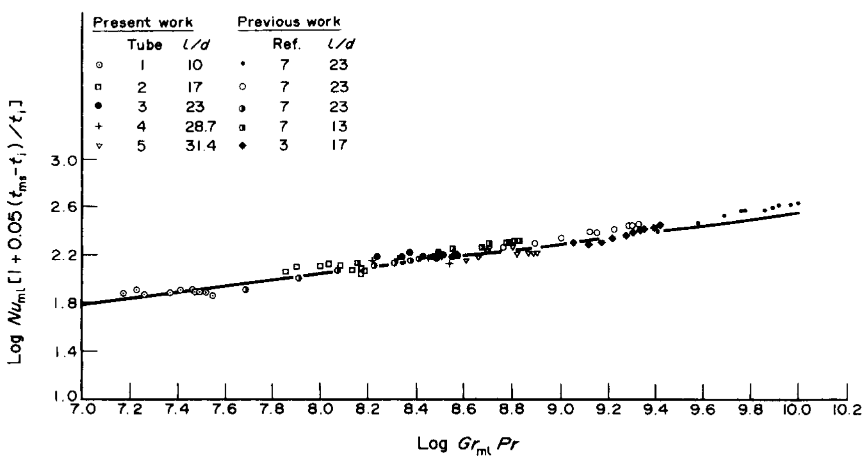

FIG. 6. Corrected values of Nu_{ml} against Gr_{ml} Pr for vertical tubes.

Table 2. Values of constant A (equation (8)) for different inclinations

θ (deg)	A
0	1.11
30	1.084
45	1.035
60	0.983
75	0.899

natural convection inside tubes was discussed by Ede [2]. Considering the results of the constant wall temperature experiments of Shen [9] on water in vertical tubes he showed that turbulence, as judged by the slope of the log Nu_{ml}/log $(Gr_{ml}\,Pr)$ line seems to have taken place at a value of $Gr_{ml}\,Pr$ between $10^{9.5}$ and $10^{11.5}$ which confirms that the present experiments were in the laminar range.

4.4. *Results of the inclined tubes*

The $Nu_{ml}/(Gr_{ml}\,Pr)$ relations given by the inclined tubes have the same general shape given by the vertical tubes. The same procedure described under Section 4.3 was therefore herein applied. All the tubes give straight line relations between $\log(Nu_{ml}f)$ and $\log(Gr_{ml}\,Pr)$ with practically the same value of n as that of the vertical tube (0.25) but with values of A depending on the inclination as given in Table 2.

It can be seen from Table 2 that the heat transfer decreases with inclination, the maximum value being at $\theta = 0°$. The rate of decrease increases slightly between $\theta = 0°$ and 45°, and more rapidly beyond that.

At $\theta = 45°$ Nu_{ml} is some 7% (whereas at $\theta = 75°$ it is some 20%) less than Nu_{ml} at $\theta = 0°$.

In the absence of data on inclined tubes the effect of θ may be compared with that reported in an investigation by Azevedo and Sparrow [10] on open-ended parallel-walled inclined channels. Their results could be represented by an equation of the form of equation (8), with the constant A independent of the angle of inclination. In the light of the present work this must be expected as the parallel-walled channel experiments were carried out in the range of θ between 0° and 45° only. Had the present experiments been carried out in this range the variation of Nu_{ml} with θ would have been concealed. As a sample the results of the tube inclined at an angle $\theta = 60°$ are shown in Fig. 7.

4.5. *General equation of all inclinations*

Introducing the relation between A and θ (in deg) equation (10) is obtained

$$Nu_{ml} = \frac{\left[1.11 - \left(\frac{\theta}{180}\right)^{1.8}\right]}{[1+0.05(t_{ms}-t_i)/t_i]}(Gr_{ml}\,Pr)^{0.25}. \quad (10)$$

This equation represents the general equation for all tubes (including the vertical tubes) in the ranges employed.

It may be noted that $Gr_{ml}\,Pr\cos\theta$ can also be used instead of $Gr_{ml}\,Pr$. In this case the values of A in Table 2 have to be divided by $(\cos\theta)^{0.25}$. The general equation for all inclinations will then be

$$Nu_{ml} = \frac{\left[1.11 + \left(\frac{\theta}{180}\right)^{2.2}\right]}{[1+0.05(t_{ms}-t_i)/t_i]}(Gr_{ml}\,Pr\cos\theta)^{0.25}. \quad (11)$$

Both equations (10) and (11) are valid in the ranges of $Gr_{ml}\,Pr$ from 1.44×10^7 to 8.85×10^8, l/d from 10 to 31.4 and θ from 0° to 75°.

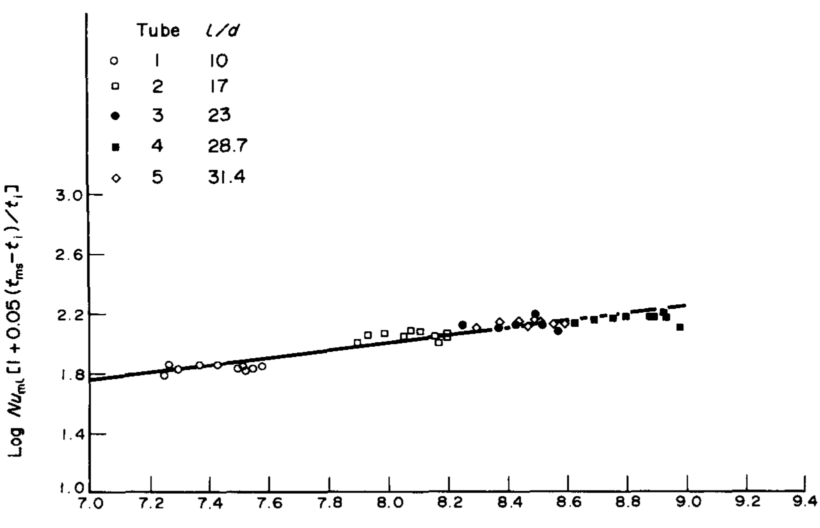

FIG. 7. Corrected values of Nu_{ml} against $Gr_{ml}\,Pr$ at $\theta = 60°$.

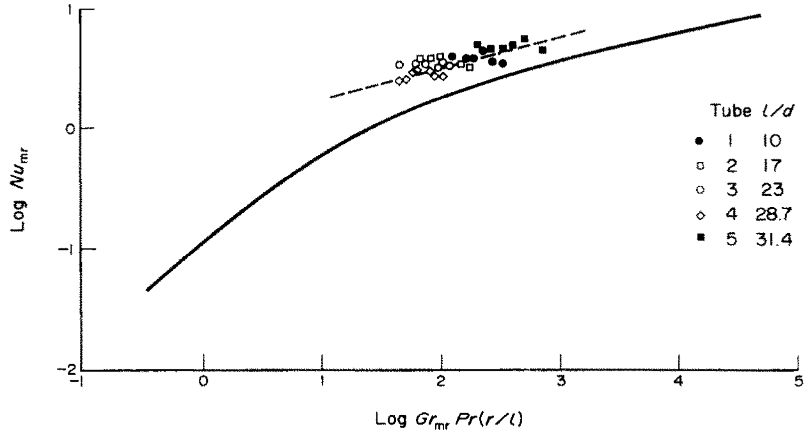

FIG. 8. Comparison with Dyer's curve.

4.6. *Effect of l/d*

One would expect Nu_{ml} to vary not only with Gr_{ml} Pr but also with l/d. However, as Fig. 6 shows, the results of all tubes are represented by the same line irrespective of the value of l/d, a fact which may be attributed to the insufficiency of the range of l/d employed to cause a significant effect on Nu_{ml}. It is of interest to note that the same result was also found in the parallel-walled channel experiments of ref. [10]. In these experiments the ratio of 'plate height/spacing between plates' varied from about 10 to 23 (which is approximately the same range as that of l/d of the present experiments).

5. COMPARISON WITH PREVIOUS WORK

A comparison of the results of the present experimental work is hereinafter made with the available theoretical and experimental work in the field of uniformly heated vertical tubes. To the authors' knowledge no work on inclined tubes is available.

(1) The present results, recalculated in terms of Nu_{mr} and Gr_{mr} $Pr(r/l)$ with physical properties taken at t_i, are compared with Dyer's theoretical curve [6] in Fig. 8. The experimental values are some 40% higher than those given by the theoretical curve which is believed to be a result of assuming physical properties independent of temperature in the theoretical study.

(2) The experimental results obtained by Al-Arabi [3] and those of Ismaeel [7], recalculated in the form of equation (9) are also plotted in Fig. 6. As can be seen they are in good agreement with the present results.

6. CONCLUSIONS

The present work was carried out in the range of Gr_{ml} Pr from 1.44×10^7 to 8.85×10^8, l/d from 10 to 31.4 and θ from 0° to 75°. It provides data in a field in which only limited information is available.

The measured heat transfer was found to vary with the angle of inclination of the tube. The maximum values were given by the vertical tube.

For all the angles of inclination employed the heat transfer could be represented by a general equation (equation (9) or (10)) in which a factor covering the variation of physical properties due to temperature gradient was employed.

In the range covered by the experiments the effect of l/d on Nu_{ml} was insignificant and the entrance length was practically constant.

Compared with the available theoretical data for the vertical tube, the experimental results were some 40% higher. In the theoretical data the physical properties were considered independent of temperature.

The available experimental data for vertical tubes were found to agree with the present results.

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CONVECTION THERMIQUE NATURELLE A LA SURFACE INTERIEURE D'UN TUBE UNIFORMEMENT CHAUFFE POUR DIFFERENTS ANGLES D'INCLINAISON

Résumé—La convection thermique naturelle par l'air à la surface intérieure de tubes verticaux ou inclinés est étudiée expérimentalement pour les domaines de $Gr_{ml}\,Pr$ entre $1,44 \times 10^7$ et $8,85 \times 10^8$ et de θ depuis 0° jusqu'à 75°. Les résultats obtenus sont unifiés par des groupes adimensionnels et comparés aux données disponibles. Le rapport longueur/diamètre de 10 à 31,4 n'a pratiquement pas d'effet sur le transfert thermique moyen pour le tube entier, dans le domaine de $Gr_{ml}\,Pr$ considéré.

DER WÄRMEÜBERGANG DURCH NATÜRLICHE KONVEKTION AN DER INNENSEITE EINES GLEICHMÄSSIG BEHEIZTEN ROHRES BEI VERSCHIEDENEN NEIGUNGSWINKELN

Zusammenfassung—Die natürliche Konvektion von Luft an der Innenseite von senkrechten und geneigten Rohren wurde experimentell im Bereich $1,44 \times 10^7 < Gr_{ml}\,Pr < 8,85 \times 10^8$ und für Neigungswinkel zwischen 0° und 75° untersucht. Die Ergebnisse werden mit Hilfe dimensionsloser Kennzahlen korreliert und mit Daten aus der Literatur verglichen. Wenn das Verhältnis von Rohrlänge zu -durchmesser zwischen 10 und 31,4 liegt, wird der Wärmeübergang im untersuchten Bereich von $Gr_{ml}\,Pr$ nicht nennenswert beeinflußt.

ТЕПЛООБМЕН НА ВНУТРЕННЕЙ ПОВЕРХНОСТИ РАВНОМЕРНО НАГРЕТОЙ ТРУБЫ ПРИ РАЗЛИЧНЫХ УГЛАХ НАКЛОНА В УСЛОВИЯХ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ

Аннотация—В диапазоне изменения значений $Gr_{ml}\,Pr$ от $1,44 \times 10^7$ до $8,85 \times 10^8$ и θ от 0° до 75° экспериментально исследована естественная конвекция в воздухе у внутренней поверхности вертикальной и наклонной труб. Полученные результаты представлены в виде критериальных зависимостей и сравниваются с известными данными. В исследованном диапазоне изменения значений $Gr_{ml}\,Pr$ возраставшее с 10 до 31,4 отношение длины трубы к ее диаметру практически в среднем не влияет на теплообмен на всей поверхности трубы.